Chp. 17: Logarithmic (Log) Functions

17.1: Inverse of Exponential Funct.

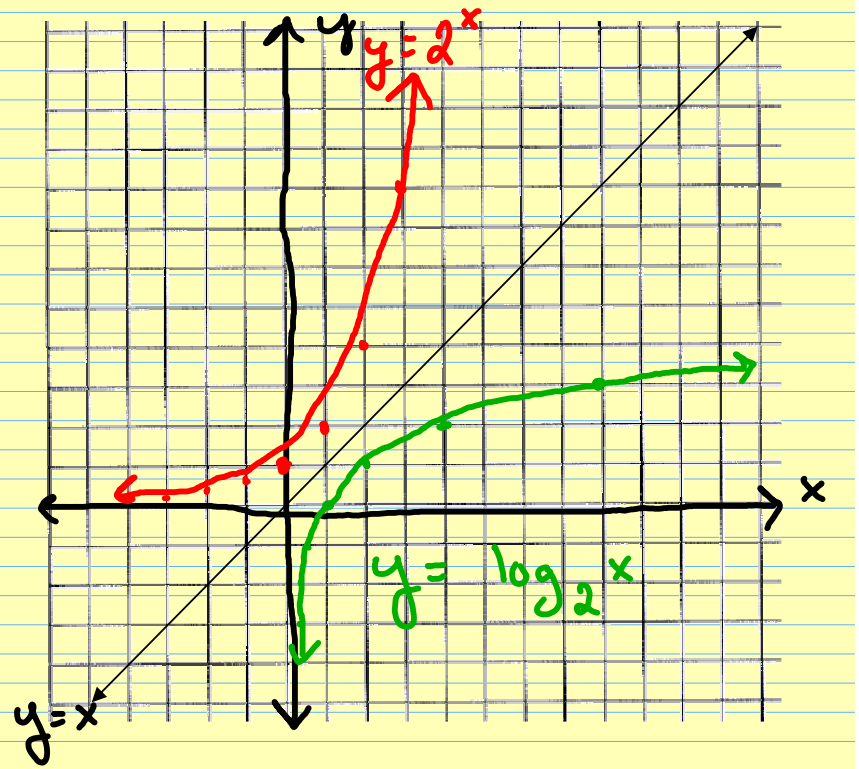
Recall: an inverse function, $f^{-1}(x)$, undoes the rule of the original function.

Inverse Graphically: reflect through $y = x$

$(x, y) \xrightarrow{r_{y=x}} (y, x)$

$f(x) = 2^x$

x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8



$r_{y=x}$

$(1/8, -3)$
$(1/4, -2)$
$(1/2, -1)$
$(1, 0)$
$(2, 1)$ $(4, 2)$ $(8, 3)$

Inverses Algebraically:

1) Switch x and y
2) Solve y

$f(x) = 2^x$

or y

$x = 2^y$

"y is the exponent of base 2 to get x."

"y is the logarithm base 2 of x."

$y = \log_2 x$

★ logarithm is simply a power

In general: the inverse of an exponential function is a logarithmic (log) function of the same base (visa-versa)

★ $y = b^x$ inverse $y = \log_b x$

$y = \log_c d$ inverse $y = c^d$

Ex What is the inverse of $y = \left(\frac{1}{3}\right)^x$?

$y = \log_{\frac{1}{3}} x$

Ex: What is the inverse of $f(x) = \log_5 x$

$f^{-1}(x) = 5^x$

$y = \log \frac{1}{3} x$

789: 2, 4, 10 - 18 all

2. a. Copy and complete the table at the right by finding, for each given value of x, the corresponding value of $y = 4^x$.

x	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
$y = 4^x$							

b. Sketch the graph of $y = 4^x$ to include the points whose ordered pairs were found in part **a**.

c. Name seven ordered pairs found in the graph of the inverse of $y = 4^x$.

d. Sketch the graph of $y = \log_4 x$ to include the points named in part **c**.

3. The table at the right lists three selected pairs of the function $y = 5^x$. Note that values of y are stated as rational approximations to the nearest tenth.

x	5^x	$= y$
$-\frac{1}{2}$	$5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$	≈ 0.4
$\frac{1}{2}$	$5^{\frac{1}{2}} = \sqrt{5}$	≈ 2.2
$\frac{3}{2}$	$5^{\frac{3}{2}} = (\sqrt{5})^3 = 5\sqrt{5}$	≈ 11.2

a. Sketch the graph of $y = 5^x$ in the interval $-1 \le x \le \frac{3}{2}$, including points where $x = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$, and $\frac{3}{2}$.

b. Sketch the graph of $y = \log_5 x$ (that is, the reflection of $y = 5^x$ in the line $y = x$).

4. a. Name five ordered pairs in the function $y = 6^x$.

b. Name five pairs in the function $y = \log_6 x$.

c. Sketch the graph of $y = \log_6 x$.

5. a. Sketch the graph of $y = 8^x$, including points for which $x = -1, -\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1$.

b. Sketch the graph of $y = \log_8 x$, including the images of the ordered pairs in part **a** under a reflection in the line $y = x$.

6. a. Sketch the graph of $y = \left(\frac{5}{2}\right)^x$ in the interval $-1 \le x \le 2$.

b. Sketch the graph of $y = \log_{\frac{5}{2}} x$ in the interval $\frac{2}{5} \le x \le \frac{25}{4}$.

c. What is the y-intercept of $y = \left(\frac{5}{2}\right)^x$?

d. What is the x-intercept of $y = \log_{\frac{5}{2}} x$?